**Sample question**: your company wants to improve sales. Past sales data indicate that the average sale was $100 per transaction. After training your sales force, recent sales data (taken from a sample of 25 salesmen) indicates an average sale of $130, with a standard deviation of $15. Did the training work? Test your hypothesis at a 5% [alpha level](http://www.statisticshowto.com/what-is-an-alpha-level/).

<http://www.statisticshowto.com/one-sample-t-test/>

<https://tc3.edu/instruct/sbrown/stat/ps03.htm>

Example 3: A chemical engineer has the following results for the active ingredient yields from 16 pilot batches processed under a retorting procedure: x bar = 32g/litre , s= 3Determine the approximate probability for getting a result this rare or rarer if the true mean yield is 30.5 grams/liter.

**Question**:  Are gender and education level dependent at 5% level of significance?  In other words, given the data collected above, is there a relationship between the gender of an individual and the level of education that they have obtained?

Here's the table of expected counts:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | High School | Bachelors | Masters | Ph.d. | Total |
| Female | 50.886 | 49.868 | 50.377 | 49.868 | 201 |
| Male | 49.114 | 48.132 | 48.623 | 48.132 | 194 |
| Total | 100 | 98 | 99 | 98 | 395 |

So, working this out, χ2=(60−50.886)2/50.886+⋯+(57−48.132)2/48.132=8.006

The critical value of χ2 with 3 degree of freedom is 7.815. Since 8.006 > 7.815, therefore we reject the null hypothesis and conclude that the education level depends on gender at a 5% level of significance.

Example

Suppose the Penn State student population is 20% PA resident and 80% non-PA resident. Then, if a sample of 100 students yields 16 PA resident and 84 non-PA resident, how 'good' do the data 'fit' the assumed probability model of 20% PA resident and 80% non-PA resident?

We can use the chi-square goodness-of-fit statistic to test the hypotheses statements:

**Null Hypothesi**s: Pr=0.2

**Alternative Hypothesis**:  Pr≠0.2

Working this out we get,

χ2=(16−20)220+⋯+(84−80)280=1

The critical value of  χ2 with 1 degree of freedom is 3.84. Since 1 < 3.84, we can not reject the null hypothesis. There is not enough evidence to conclude that the data don't fit the assumed probability model at 5% level of significance.  In other words, the students that were randomly selected in this example did resemble the probability distribution that was specified.

<https://onlinecourses.science.psu.edu/statprogram/node/158>

## **Example 2: A Fair Gamble**

Many casinos use card-dealing machines to deal cards at random. Occasionally, the machine is tested to ensure an equal likelihood of dealing for each suit. To conduct the test, 1,500 cards are dealt from the machine, while the number of cards in each suit is counted. Theoretically, 375 cards should be dealt from each suit. As you can see from the results in our table, this is not the case:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Spades** | **Diamonds** | **Clubs** | **Hearts** |
| Observed | 402 | 358 | 273 | 467 |
| Expected | 375 | 375 | 375 | 375 |

We can use chi square to determine if the discrepancies are significant. If the discrepancies are significant, then the game would not be fair. Measures would need to be taken to ensure that the game is fair.

**2.  Pre-school Attendance and Pre-algebra Achievement**  
(these are contrived data, based on a real study)  
In these times of educational reform, attention has been focused on pre-school for all children. Since many districts are facing budget cuts, funding pre-school programs may impact other offerings. Before making their recommendations, administrators in a large urban district take a random sample of 50 seventh graders and compare the pre-algebra achievement levels of those who attended pre-school and those who did not. If achievement is independent of attending pre-school then the proportions at each level should be equal. Use the counts in the frequency table to determine if there is an association between attending pre-school and pre-algebra achievement.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Below grade level | At grade Level | Advanced |
| Pre-school | 8 | 6 | 6 |
| No Pre-school | 6 | 15 | 9 |

**Practice Problems**

**1. Jury Selection (adapted from the Freedman, Pisani, Purves classic text)**  
One study of grand juries in Alameda County, California, compared the demographic characteristics of jurors with the general population, to see if jury panels were representative. The results for age are shown below. The investigators wanted to know if the 66 jurors were selected at random from the population of Alameda County. (Only persons over 21 and over are considered; the county age distribution is known from Public Health Department data.) The study was published in the UCLA Law Review.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Age | Count-wide % | # of jurors observed | # of jurors expected | (O-E) | (O-E)2/E |
| 21-40 | 42% | 5 |  |  |  |
| 41-50 | 23% | 9 |  |  |  |
| 51-60 | 16% | 19 |  |  |  |
| over 60 | 19% | 33 |  |  |  |
| Total | 100% | 66 |  |  |  |

Do we have evidence that grand juries are selected at random for the population of Alameda County?

**3. Evaluating Textbooks**  
Does the new math program improve student performance?   
Suppose you take a random sample of 20 students who are using a new algebra text which features group work and unit summaries and a second sample of 30 students who are using a more traditional text. You compare student achievement on the state test given to all students at the end of the course. Use the frequency table to determine if the proportions from each group are equal at each performance level.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Below grade level | At grade level | Advanced |
| New text | 8 | 6 | 6 |
| Old text | 6 | 15 | 9 |

**1.  Jury Selection**  
*Checking Conditions*: We have a random sample; the sample size is less than 10% of the county population; all expected cells are larger than 5, so a Chi-squared test is appropriate.  
*Stating Hypotheses*:  
Ho:  For each age group, the proportion of jurors is consistent with the county proportion.  
Ha:  The proportion of jurors for at least one age group is inconsistent with the county proportions.

*Calculating the test statistic*:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Age | Observed | Expected | (O-E) | (O-E)2/E |
| 21 to 40 | 5 | 0.42(66) = 27.72 | -22.72 | 18.62 |
| 41 to 50 | 9 | 15.18 | -6.18 | 5.55 |
| 51 to 60 | 19 | 10.56 | 8.44 | 6.75 |
| 61 or over | 33 | 12.52 | 20.46 | 33.38 |
| *totals* | 66 | 66.00 | 0 | **61.2556** |

Chi-squared statistic = 2.516 and df = 3, p-value is almost 0.

*Writing a conclusion in context*:  
It is almost impossible for a jury to differ this much from the county age distribution by chance. Our sample provides evidence that grand juries in Alameda County are not selected at random.  
(In a footnote, the authors tell us that grand juries are nominated by judges, who prefer older jurors.)

**2.  Pre-school Attendance and Pre-algebra Achievement**  
We use a Chi-squared test since we have one random sample,  the sample size is less than 10% of the population of all 7th graders in the district, and each expected cell count is large enough (greater than 5).

Ho:  Pre-algebra achievement is independent of pre-school attendance.  
Ha:  There is a relationship between Pre-algebra achievement and pre-school attendance

|  |  |  |  |
| --- | --- | --- | --- |
| *Expected counts* | Below grade level | At grade level | Advanced |
| Pre-school | 5.6 | 8.4 | 6 |
| No Pre-school | 8.4 | 12.6 | 9 |

Chi-squared =  2.85 with p-value = .239, much larger than alpha = .05.

Since our p-value is so high, our sample does not provide significant evidence that pre-algebra achievement is related to pre-school attendance. This study alone would not support funding pre-school education for all students in the district.

**3.  Evaluating Textbooks**  
We use a Chi-squared test since we have two random samples which were independently chosen; the sample size is less than 10% of the population of all algebra students in the district; and each expected cell count is large enough (greater than 5).

Ho:  At each proficiency level, the proportions are the same for students who used the new text and those who used the traditional text.  
Ha: There is a difference in the proportions for students who used the new text and those who used the traditional text.

|  |  |  |  |
| --- | --- | --- | --- |
| *Expected counts* | Below grade level | At grade level | Advanced |
| Pre-school | 5.6 | 8.4 | 6 |
| No Pre-school | 8.4 | 12.6 | 9 |

Chi-squared =  2.85 with p-value = .239, much larger than alpha = .05.  
Since our p-value is so large, our sample provides no significant evidence that the proportions at each level are different for students who used the new or traditional algebra text. On the basis of this study, we would probably not recommend new textbooks.

**4.  Summary**  
a)  In Jury Selection, we examine how well the sample fit our model or its goodness-of-fit.  
b)  For the Pre-school and Pre-algebra, each student in our only sample is classified by two attributes. If the variables are independent, the observed counts will be consistent with the expected counts. We want to know if those attributes are related or independent.   
c)  In the Textbooks problem, we select students in two samples and we examine a single attribute to see if the proportions are equal or homogeneous for each group.  
Although the mechanics for a test of independence and for a test of homogeneity are the same, the methods for selecting the samples differ. We ask different questions. “Are the attributes independent?” rather than “Are the groups homogeneous (or alike) with respect to this attribute?”

2. Children of three ages are asked to indicate their preference for three photographs of adults. Do the data suggest that there is a significant relationship between age and photograph preference? What is wrong with this study? [Chi-Square = 29.6, with 4 d.f.: p<0.05].

                                                                        Photograph:

                                                            A                      B                      C

            Age of child:      5-6 years:          18                     22                     20

                                    7-8 years:            2                    28                     40

                                    9-10 years:        20                     10                     40

http://users.sussex.ac.uk/~grahamh/RM1web/STPROB5new.htm

**Research Methods 1: Statistics Problem-Sheet 5: Chi-Square:**

            1. A sample of 100 voters are asked which of four candidates they would vote for in an election. The number supporting each candidate is given below:

                        Higgins             Reardon            White                Charlton

                        41                     19                     24                     16

            Do the data suggest that all candidates are equally popular? [Chi-Square = 14.96, with 3 d.f.: p<0.05].

            2. Children of three ages are asked to indicate their preference for three photographs of adults. Do the data suggest that there is a significant relationship between age and photograph preference? What is wrong with this study? [Chi-Square = 29.6, with 4 d.f.: p<0.05].

                                                                        Photograph:

                                                            A                      B                      C

            Age of child:      5-6 years:          18                     22                     20

                                    7-8 years:            2                    28                     40

                                    9-10 years:        20                     10                     40

            3. A study of conformity using the Asch paradigm involved two conditions: one where one confederate supported the true judgement, and another where no confederate gave the correct  response

                                    Support:                        No Support:

            Conform:           18                                40

            Not Conform:     32                                 10

            Is there a significant difference between the "support" and "no support" conditions in the frequency with which individuals are likely to conform? [Chi-Square = 19.87, with 1 d.f.: p<0.05. *OR: Chi-Square = 18.1.* *See the comment at the end of this handout*].

            4. A researcher is interested in sex differences in social play in cats. She observes sixteen kittens (eight male and eight females), and records the sex of the participants in 500  play-bouts, taking a note of the sex of the animal initiating a bout and the sex of the animal who responds to the initiation. Are there sex-differences in the kittens' play? Perform a Chi-Square test on these data. Is there anything wrong with doing this test? [Chi-Square = 3.97, with 1 d.f.: not significant. *OR: Chi-Square = 3.58. See the comment at the end of this handout*].

                                                                             respondent

                                                                        male                 female

            initiator of bout:             male:                200                   150

                                                female:              100                     50

            5. We want to test whether short people differ with respect to their leadership qualities (Genghis Khan, Adolf Hitler and Napoleon were all stature-deprived, and how many midget MP's are there?) The following table shows the frequencies with which 43 short people and 52 tall people were categorised as "leaders", "followers" or as "unclassifiable". Is there a relationship between height and leadership qualities? [Chi-Square = 10.71, with 2 d.f.: p<0.01].

            6. The Government decide to find out how many people use various kinds of transport to commute from Brighton to London each day. They go to Victoria train station, and ask a random sample of 80 individuals what type of transport they use. Here are the raw data, coded as follows: "1" if the person uses a car; "2" if they use a train; "3" if they walk; and "4" if they fly. The null hypothesis is that all four forms of transport are used equally frequently by the general commuting public.

            (a) Draw a frequency histogram of these data.

            (b) Perform a Chi-Square Goodness of Fit test on these data.

            1,4,2,3,4,4,3,4,4,2          2,2,1,4,3,3,2,2,2,2

            3,2,4,4,3,2,1,1,2,3          4,4,3,3,2,1,3,2,4,3

            2,1,2,2,2,1,2,3,2,4          3,2,2,3,4,2,3,4,2,2

            2,1,2,1,2,2,2,3,4,3          2,2,1,3,2,2,2,1,2,2

            What problems are there with this study?

**Chi-Square in the 1 d.f. case, and the use of Yates' Correction:**

If you have only 1 d.f., as in the case of a 2x2 contingency table, some textbooks suggest that you apply what's known as "Yates' Correction" to the Chi-Square formula. When the d.f. are very small, (and you can't get much smaller than 1!), the Chi-Square sampling distribution becomes increasingly distorted. As a consequence, the obtained value of Chi-Square tends to overestimate the "true" discrepancy between the observed and expected frequencies. Yates' correction is a simple bodge that makes the formula produce a lower value of Chi-Square than it otherwise would do. This makes the Chi-Square test more "conservative": i.e., it makes it harder to get a statistically significant result. Here is the "corrected" formula:

http://users.sussex.ac.uk/~grahamh/RM1web/STPROB5new_files/image002.gif

            In English, this means you do the following:

            (a) Get the absolute value (i.e., ignore the sign) of each observed frequency minus its accompanying expected frequency. (The vertical lines mean "ignore the sign of").

            (b) From each of these absolute values, subtract 0.5.

            (c) Divide each value in step (b) by the associated expected frequency.

            (d) Add together all of the results of step (c).

            (e) Finally, look up the value of Chi-Square in the usual way.

            Statisticians are divided about the need for Yates' correction: some say you should use it, others say it doesn't matter, and some say it makes things worse! Consequently, the answers to the problems on this sheet have been calculated both ways. The normal-type answers are what you would get if you did not use Yates' correction, and the italic-type answers are what you would get if you did. Either is acceptable, as long as you make it clear whether or not you used Yates' correction. Remember - if you do use it, it is *only* to be used when you have just one degree of freedom.

**First-year Research Methods: Worked Solutions to Problem Sheet 5:**

**Question 1:**

A Chi-Squared Goodness-of-Fit test is appropriate here. The null hypothesis is that there is no preference for any of the candidates: if this is so, we would expect roughly equal numbers of voters to support each candidate. Our expected frequencies are therefore 100/4 = 25 per candidate.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **O** | **41** | **19** | **24** | **16** |
| **E** | **25** | **25** | **25** | **25** |
| **(O-E)** | **16** | **-6** | **-1** | **-9** |
| **(O-E)2** | **256** | **36** | **1** | **81** |
| **(O-E)2**  **---------**  **E** | **10.24** | **1.44** | **0.04** | **3.24** |

Adding together the last row gives us our value of 2 :

       (O - E)2

 -----------------   = 10.24+ 1.44 + 0.04 + 3.24 = **14.96**, with 4 - 1 = 3 degrees of freedom.

           E

            The critical value of Chi-Square for a 0.05 significance level and 3 d.f. is 7.82. Our obtained Chi-Square value is bigger than this, and so we conclude that our obtained value is unlikely to have occurred merely by chance. In fact, our obtained value is bigger than the critical Chi-Square value for the 0.01 significance level (13.28). In other words, it is possible that our obtained Chi-Square value is due merely to chance, but highly unlikely: a Chi-Square value as large as ours will occur by chance only about once in a hundred trials. It seems more reasonable to conclude that our results are not de to chance, and that the data do indeed suggest that voters do not prefer the four candidates equally.

**Question 2:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | | **photograph:** | | |  |
| **age of child:** | **A:** | | **B:** | **C:** | **row totals:** |
| **5-6 years** | **18** | | **22** | **20** | **60** |
| **7-8 years** | **2** | | **28** | **40** | **70** |
| **9-10 years** | **20** | | **10** | **40** | **70** |
| **column totals:** | **40** | | **60** | **100** | **200** |
|  |  |  |  |  |  |

(a) Work out the row, column and grand totals (as shown in the shaded parts of the table, above).

            (b) Work out the expected frequencies, using the formula:

                              (row total \* column total)

            E =           --------------------------------------

                                      grand total

            For each cell of the above table, this gives us:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **O:** | **18** | **22** | **20** | **2** | **28** | **40** | **20** | **10** | **40** |
| **E:** | **12** | **18** | **30** | **14** | **21** | **35** | **14** | **21** | **35** |

Next, work out (O - E):

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **(O-E):** | **6** | **4** | **-10** | **-12** | **7** | **5** | **6** | **11** | **5** |

Square each of these, to get (O - E)2:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **(O - E)2:** | **36** | **16** | **100** | **144** | **49** | **25** | **36** | **121** | **25** |

Divide each of the above numbers by E, to get  (O - E)2 / E:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **(O - E)2**  **----------**  **E** | **3** | **0.89** | **3.33** | **10.29** | **2.33** | **0.71** | **2.57** | **5.76** | **0.71** |

Chi-squared is the sum of these:

            2 = **29.60**.

            d.f. = (rows - 1) \* (columns - 1) = 2 \* 2 = 4.

            The critical value of Chi-Square in the table for a 0.001 significance level and 4 d.f. is 18.46. Our obtained Chi-Square value is bigger than this: therefore we have a Chi-Square value which is so large that it would occur by chance only about once in a thousand times. It seems more reasonable to accept the alternative hypothesis, that there is a significant relationship between age of child and photograph preference.

**Question 3:**

Here we have a 2x2 contingency table. Chi-Square is the appropriate test to use, but since we have 1 d.f., we will modify the formula to include "Yates' correction for continuity".

|  |  |  |  |
| --- | --- | --- | --- |
|  | **support** | **no support** | **row totals:** |
| **conform:** | **18** | **40** | **58** |
| **not conform:** | **32** | **10** | **42** |
| **column totals:** | **50** | **50** | **100** |

(a) Calculate the row, column and grand totals.

            (b) Calculate the expected frequency for each cell of the table, by multiplying together the appropriate row and column totals and then dividing by the grand total.

            (c) Subtract each expected frequency from its associated observed frequency; but then apply Yates' correction, by subtracting 0.5 from the absolute value of each O-E value. (The vertical bars in the formula mean "ignore any minus signs").

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **O:** | **18** | **40** | **32** | **10** |
| **E:** | **29** | **29** | **21** | **21** |

Next, work out (O - E):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **(|O-E|- 0.5):** | **10.5** | **10.5** | **10.5** | **10.5** |

Square each of these, to get (O - E)2:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **(|O-E|- 0.5)2:** | **110.25** | **110.25** | **110.25** | **110.25** |

Divide each of the above numbers by E, to get  (O - E)2 / E:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **(O - E)2**  **-----------**  **E** | **3.80** | **3.80** | **5.25** | **5.25** |

Chi-squared is the sum of these:

            2 = **18.10.**

            d.f. = (rows - 1) \* (columns - 1) = 1 \* 1 = 1.

            Our obtained value of Chi-Squared is bigger than the critical value of Chi-Squared for a 0.001 significance level. In other words, there is less than a one in a thousand chance of obtaining a Chi-Square value as big as our obtained one, merely by chance. Therefore we can conclude that there is a significant difference between the "support" and "no support" conditions, in terms of the frequency with which individuals conformed.

**Question 4:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | | **respondent:** | |  |
| **initiator:** | **male** | | **female** | **row totals:** |
| **male** | **200** | | **150** | **350** |
| **female** | **100** | | **50** | **150** |
| **column totals:** | **300** | | **200** | **500** |
|  |  |  |  |  |

(a) Calculate the marginal totals and expected frequencies.

            The expected frequencies are obtained by multiplying the row total by the column total, and then dividing by the grand total. For example, the expected frequency for male-male play is (350\*300)/500 = 210; the expected frequency for male-female play is (150\*200)/500 = 60; and so on. Expected frequencies are shown in brackets below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | | **respondent:** | |  | |
| **initiator:** | **male** | | **female** | | **row totals:** |
| **male** | **200 (210)** | | **150 (140)** | | **350** |
| **female** | **100 (90)** | | **50 (60)** | | **150** |
| **column totals:** | **300** | | **200** | | **500** |
|  |  |  |  |  |  |

NB: the expected frequencies should add up to the same Grand Total (500 in this case) as the observed frequencies!

            (b) Since we have a 2x2 table (and hence 1 d.f.) we will work out Chi-Square incorporating Yates' correction for continuity. For each cell in the table, work out:

http://users.sussex.ac.uk/~grahamh/RM1web/STPROB5new_files/image004.gif

            This gives us the following numbers:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | | **respondent:** | |  |
| **initiator:** | **male** | | **female** | |
| **male** | **0.4298** | | **0.6446** | |
| **female** | **1.0028** | | **1.5042** | |
|  |  |  |  |  |

(c) Adding these together gives us our value of Chi-Square: **3.581**, with 1 d.f. This is smaller than the critical value of Chi-Square for 1 d.f. at the 0,.05 significance level, which is 3.841. We would therefore conclude that the frequencies with which male and female animals play together in opposite-sex and liked-sexed pairs do not differ from those that we would expect by chance.

            In practice, there is a problem with this analysis which invalidates the Chi-Square test: we have 500 observations from only 16 animals, which means that each animal must have contributed more than observation to the total. This means that the observations are not independent - thus violating an important assumption for the use of the Chi-Square test.

**Question 5:**

            Expected frequencies are in brackets:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | | **height:** | | |  |
|  | **short** | | **tall** | **row totals:** | |
| **leader:** | **12 (19.92)** | | **32 (24.08)** | **44** | |
| **follower:** | **22 (16.29)** | | **14 (19.71)** | **36** | |
| **unclassifiable:** | **9   (6.79)** | | **6   (8.21)** | **15** | |
| **column totals:** | **43** | | **52** | **95** | |
|  |  |  |  |  |  |

Chi-Square  = 3.146 + 2.602 + 1.998 + 1.652 + 0.720 + 0.595 = **10.712**, with 2 d.f.

            10.712 is bigger than the tabulated value of Chi-Square at the 0.01 significance level. We would conclude that there seems to be a relationship between height and leadership qualities. Note that we can only say that there is a relationship between our two variables, not that once causes the other. There could be all kinds of explanations for such a relationship.

**Question 6:**

            Our 80 subjects fall into four transport categories as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| **type of transport:** | | | |
| **car (1)** | **train (2)** | **walk (3)** | **fly (4)** |
| **11** | **36** | **18** | **15** |

Our null hypothesis is that each form of transport is used equally frequently. Therefore our expected frequencies are (80/4) = 20, 20, 20 and 20. The appropriate analysis is a Chi-Squared Goodness-of-Fit test.

            Take each observed frequency; subtract 20; square the result; and then divide by 20. This gives 4.05, 12.80, 0.20 and 1.25. Adding these values together gives us our value of Chi-Square: 18.30, with 3 d.f., p<0.001. In other words, our obtained frequencies are markedly different from those that we would expect to obtain by chance. Strictly speaking, this is all we can say. However, "eyeballing" the data suggests that more people travel by train than we would expect, and fewer by car (well, these are fictional data!)

            The problem with this study lies not in the statistical analysis, but in the way that the data were obtained: standing in a train station is likely to bias the sample in favour of people travelling by train.

Which is the best coffee (most cups ordered):

Blue-Label   Green-Label     Red-Label   
            13              1              5   
              4              1              2   
            10              2              2   
            13              2              2   
            11              2              6   
              3              4              4